Gripping Parts at Concave Vertices*

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1 Abstract

A simple gripper with two vertical cylindrical jaws can make contact with external or internal concavities in polygonal and polyhedral parts to align and grip parts in form closure. Such grippers are inexpensive, lightweight, and their small footprint facilitates access and insertion for industrial applications.

We refer to this as a **v-grip.** *We begin by defining 2D v-grips, where a pair of frictionless point jaws makes contact with a pair of polygonal part concavities to achieve form-closure. We define a v-grip quality metric based on the maximum possible change in the part's orientation when jaw position is relaxed infinitesimally. For a polygonal part with polygonal holes, we give an algorithm for computing and ranking 2D v-grips. We also extend the definition to jaws with non-zero radii. In 3D, v-grips are achieved with a pair of frictionless vertical cylinders. We define* **3D v-grips** *and give a numerical algorithm for computing all 3D v-grips of a polyhedral part.*

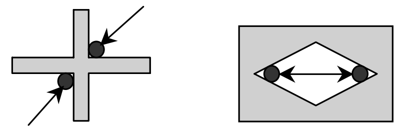
If n is the number of vertices that describe the part and k is the number of concave vertices, we can compute all 2D v-grips in $O(n+k^2)$ *time. Computing offsets for jaws with non-zero radii takes* $O(n \log n)$ *time. A ranked list of 2D v-grips based on the quality metric can be computed in an additional* $O(k^2 \log k)$ *time. We find all 3D v-grips in* $O(n^3 k^2)$ *time. A Java implementation of the 2D v-grip algorithm is available online for testing.*

2 Introduction

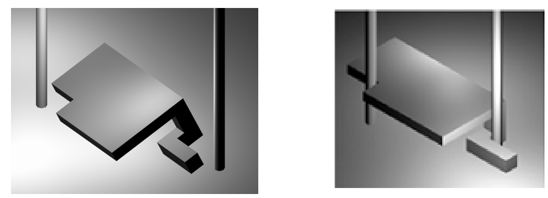
As illustrated in figure 1, parts can be gripped with two cylindrical jaws by contracting or expanding the jaws toward a pair of concavities. One frictionless contact at a concave vertex can generate forces equivalent to two contacts at part edges. In this paper, we formalize conditions for, and study the properties of *v-grips* for rigid polygonal parts with jaws of zero radii. A v-grip is *contracting* if the jaws move towards each other and *expanding* if the jaws move away from each other. We analyze v-grips using a distance function. We define a new quality metric based on the maximum possible change in the part's orientation when jaw position is relaxed infinitesimally This can be computed efficiently to rank v-grips and is consistent in most cases with physical intuition.

* This work was supported in part by the National Science Foundation under DMI-0010069. Research funding was also provided by Adept Technology, and California State MICRO Grant 00-032.

We then consider v-grips in 3D where a given polyhedral part rests on a horizontal planar work-surface under the influence of gravity.



(a) Contracting v-grip (b) Expanding v-grip



(c) Initial 3D part orientation (d) Final 3D v-grip

Figure 1: Examples of v-grips in 2D and 3D.

3 Related Work

Bicchi and Kumar [2] and Mason [11] summarize research in robot grasping. Grasps can be classified as force or form-closure. Form-closure occurs when any neighboring configuration of the part results in collision with an obstacle. Force-closure occurs if any external wrench can be resisted by applying suitable forces at the contacts [11,21].

Gripper contacts have been modeled as frictional points, frictionless points or soft contacts [24]. [19] and [26] prove that 4 and 7 frictionless point contacts are necessary to establish form closure in the plane and in 3D respectively. [12] and [10] proved that 4 and 7 point contacts suffice. These lower bounds assume that contacts occur at smooth boundaries of the part where a unique normal is well defined.

Van der Stappen et al [29] give an efficient algorithm to compute all *Nguyen regions*: sets of placements of four frictionless point contacts on a polygonal part that ensure form-closure. Given a set of four edges, they show how to compute critical contact placements in constant time. The time complexity of their algorithm is bounded by the number of such sets, $O(n^4)$ in the worst case. A jaw in contact with a vertex is considered as two jaws in contact with the neighboring edges.

Rimon and Burdick [21] and [22] were the first to identify and introduce the notion of second order force closure. First order immobility occurs if every direction of motion in C-space has a negative component along some outward normal to a C-obstacle in contact with the part. For second order mobility, every trajectory approximated up to its second order derivative in the

Taylor expansion results in a decrease in distance from some C-obstacle in contact with the part. [20] shows that generic planar parts can be immobilized (second-order) with three frictionless contacts if they are placed with infinite precision. Ponce et al [17] give an algorithm to compute such configurations. [20] also gives a sufficient condition for immobility using two fingers when contacting jaws have the necessary curvature. Their analysis is for a smooth body with a unique normal at the point of contact.

For force-closure grasps with friction, Faverjon and Ponce [6] compute grasps for curved parts using 2 parallel cylindrical jaws. Chen and Burdick [5] use two point contacts to grip parts at antipodal points. Mirtich and Canny [13] study how 2D and 3D parts can be grasped by 2 and 3 fingers respectively. They compute optimal force-closure grasps with frictional contacts, optimality being measured in terms of the ratio of magnitudes of contact forces to external wrenches. Tung and Kak [28] give an algorithm to generate two point frictional force-closure grasps at edges of polygonal parts. [25] show how such grasps can be optimized to resist slip about the part's center of mass when it is lifted.

Park and Starr [14] consider force-closure grasps of a polygonal part created using a 3-fingered robot hand by considering combinations of edges and vertices. At convex vertices, they use specially shaped fingers to grasp the part. [18] computes grasps for polygons using 3 fingers with friction.

Lynch [9] formalizes conditions for toppling objects considering contact friction, location, and motion. Zhang and Goldberg [30] give a numeric algorithm to design jaws for aligning parts in the vertical plane. They use trapezoidal jaw modules that maximize contact between the jaws and the part by combining analysis of toppling, jamming, liftoff, accessibility and form-closure.

Berretty et al [1] describe a method to orient parts by pulling with one cylindrical jaw that generalizes the algorithm described in [7]. The stable positions when pulling with the jaw occur only when the jaw is in a concavity.

Sugar and Kumar [27] give an excellent review of grasp quality metrics and propose frame-invariant quality metrics based on the grasp stiffness matrix. We define a kinematic metric based on sensitivity of the part's orientation when the jaws' position is infinitesimally relaxed.

Rimon and Blake [23] give a method to find *caging grasps*, configurations of jaws that constrain parts in a bounded region of C-space such that actuating the gripper results in a unique final configuration. They consider the opening parameter of the jaws as a function of their positions and use stratified Morse theory to find caging grasps. They state without proof that equilibrium grasps occur only at extrema of the opening parameter of the gripper.

In 1996, Plut and Bone [15] and [16] proposed inside-out and outside-in grips using two or more frictionless point contacts at linear or curved part edges. They gave an algorithm for finding such grips where the distance between contacts is at an extremum. We extend and refine their results for 2 point grips, showing that the extremum is a necessary but not sufficient condition for expanding and contracting v-grips. We then give an efficient algorithm for computing them.

4. Problem Definition

We begin by defining **v-grips** in the plane. Given a planar projection of the part, we want to find and rank all available v-grips. We assume that the projection of the part onto the horizontal plane is rigid and can be defined by a polygonal boundary and polygonal holes. All contacts are frictionless. We initially assume both jaws have zero radius.

Let v_a and v_b be two concave vertices. The unordered pair $\langle v_a, v_b \rangle$ is an expanding or contracting v-grip if jaws placed at these vertices will provide frictionless form-closure of the part.

Input: Vertices of polygons representing part boundary and holes, in counter-clockwise order, and jaw radius.

Output: A list (possibly empty) of all v-grips sorted by quality measure.

5. Test for form-closure

The key to our algorithm is a constant-time test for form-closure. We consider a pair of concave vertices $\langle v_a, v_b \rangle$. Let v_{x-1} and v_{x+1} be the vertices adjacent to v_x. Let $\mathbf{u}_{x-1}$ be the unit vector from v_x to v_{x-1}, and $\mathbf{u}_{x+1}$ the unit vector from v_x to v_{x+1}. Let $\mathbf{u}_{xy}$ be the unit vector from v_x to v_y.

We construct normals at v_a, to both edges bordering v_a. This splits the plane into 4 regions (see figure 2). We number these I to IV. We do a similar construction with v_b.

Theorem 1: $\langle v_a, v_b \rangle$ is an *expanding* v-grip if and only if v_a lies strictly in region I of vertex v_b, and v_b lies strictly in region I of vertex v_a.

Theorem 2: $\langle v_a, v_b \rangle$ is a *contracting* v-grip if and only if either:

(1) v_a lies in region IV of vertex v_b, and v_b lies in region IV of vertex v_a, at least one of them strictly,

or

(2) $\mathbf{u}_x.\mathbf{u}_y = -1$ and $\mathbf{u}_x.\mathbf{u}_{ab} = \mathbf{u}_y.\mathbf{u}_{ab} = 0$ for at least one set of values of (x, y) = (a±1, b±1), and the jaws approach from outside the region between the parallel lines (see figure 3).

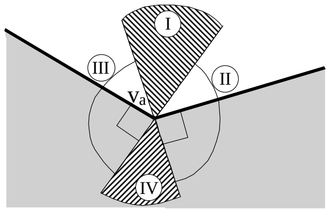


Figure 2: Two normals at a concave vertex partition the plane into 4 regions that define v-grips.

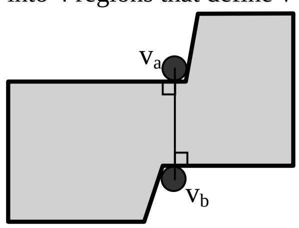


Figure 3: Example where the second condition in Theorem 2 holds.

5.1. Proof of Theorem 1

Let P represent part perimeter parameterized by arclength s. Let s_a and s_b represent the positions of the jaws on P. Following [3] and [23], we express the distance between the jaws as σ: $P \times P \to R^+$, a function of (s_a, s_b). The σ(s_a, s_b) surface is positive except when it touches the plane along the diagonal s_a=s_b (where it is 0), as these points represent coincident jaws. The s_a-s_b plane can be partitioned into rectangles whose sides are equal in length to the sides of the polygon. In each of these regions, the distance function is defined by a quadratic expression.

To prove Theorem 1, we prove that the following 4 statements are equivalent:

A: v_a and v_b are concave and they each lie in the other's region I.

B: σ(s_a, v_b) is a strict local maximum at s_a=v_a, and σ(v_a, s_b) is a strict local maximum at s_b=v_b.

C: σ(s_a,s_b) is a strict local maximum at s_a=v_a and s_b=v_b.

D: <v_a, v_b> is an expanding v-grip for the part.

B⇔A: This is clearly seen since the shortest distance from a point to a line is along the normal to the line (figure 4).

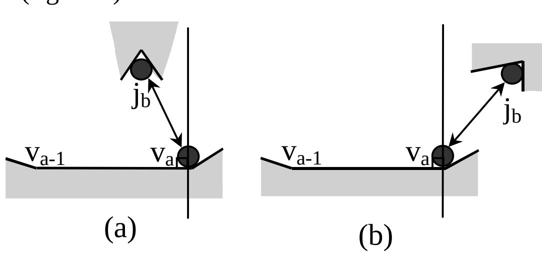


Figure 4: j_bv_a is a strict local maximum (a) or a local minimum (b) for s_a in $v_{a-1}v_a$.

C⇒B follows from the definitions.

B⇒C: Assume B. Since B⇔A, A is true. Therefore, v_b lies strictly in region I of v_a. Hence, there exists a small region, say a circle of radius ε (a small length) around v_b, which also lies completely in region I (figure 5).

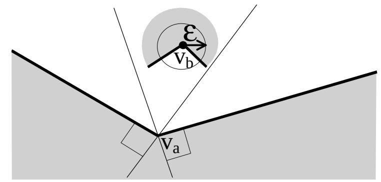


Figure 5: σ(v_a,s_b) is a local maximum of σ(s_a,s_b) for any s_b in the neighborhood of v_b.

Consider any v'_a in P, within ε from v_a, and v'_b in P within ε from v_b. Since v_a is in v_b's region I, σ(v_a, s_b) is a local maximum at s_b=v_b. Therefore, $v_av_b > v_av'_b$. Since v'_b also lies in v_a's region I, $v_av'_b > v'_av'_b$. Thus, $v_av_b > v'_av'_b$. □

Therefore, C⇔B.

C⇒D: Assume C is true and D is false. Since A⇔C, A is true. Since σ(v_a, v_b) is a local maximum and D is false, the part is not in form-closure. This means that there exists a neighboring point in C-space that does not result in collision. In other words, the part can be displaced infinitesimally. Since C is true, at least one jaw must break contact with the part in the new configuration.

If both jaws break contact, we can move the part along the directions $\pm\mathbf{u}_{ab}$ till contact occurs as both vertices are concave and hence have an angle of less than 180^o from the direction of the jaws' approach. As a result, movement in at least one of two opposite directions results in contact. From this position, we can slide the part along the contact edge moving the vertex towards the jaw, till contact occurs with the other jaw or till the vertex is at the jaw. Since v_av_b is a strict maximum, the vertex has to be reached.

However, since A is true, $\mathbf{u}_{ab}$ is at acute angles to $\mathbf{u}_{a-1}$ and $\mathbf{u}_{a+1}$, and $\mathbf{u}_{ba}$ is at acute angles to $\mathbf{u}_{b-1}$ and $\mathbf{u}_{b+1}$. Therefore, when the vertex reaches the jaw, the other jaw would collide with the interior of the part: thus the part cannot move and is in form-closure. □

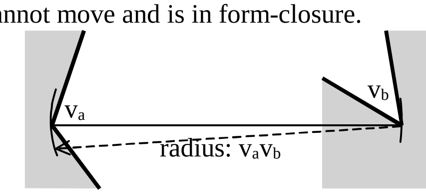


Figure 6: The edges are at acute angles to v_av_b.

D⇒C: Assume D is true and C is false. Then, σ(v_a, v_b) is not a local maximum. Either it is a strict local minimum or it is not a strict local extremum. If v_av_b is a strict local minimum it can be shown that <v_a, v_b> is a contracting v-grip, and hence D cannot be true. If v_av_b is not a strict extremum, then by the continuity of s, the part can move along the contour {(s_1, s_2) | σ(s_1, s_2) = σ(v_a, v_b)}. This contradicts D. Therefore C is true. □

Thus, D⇔C, completing the proof for theorem 1. We can prove Theorem 2 similarly. The second condition in Theorem 2 arises due to the limiting case where vertex lies on the boundary of region IV.

6. Quality Metric

We can compare v-grips based on how much the part can rotate when the jaws are relaxed infinitesimally. We define a measure of the sensitivity of the grip to such infinitesimal disturbances.

Given a v-grip $\langle v_a, v_b \rangle$, let $l = \sigma(v_a, v_b)$. If the distance between the jaws changes by Δl, let $\Delta\theta$ be the maximum angle the part can rotate. Clearly, $\Delta\theta$ depends on Δl. We consider the ratio $\Delta\theta/\Delta l$, which for infinitesimal changes becomes $d\theta/dl$. We rank parts based on $|d\theta/dl|$: smaller ratios correspond to more robust grips. It can be shown that the maximum $|d\theta|$ occurs when both jaws are in contact with the part with one of them at a vertex.

To derive an expression for $|d\theta/dl|$, we consider one edge at an angle ϕ to $v_a v_b$. Using the sine rule,

$$(l-\Delta l)/(\sin\phi) = l / (\sin(\phi + \Delta\theta))$$

If we neglect second order terms, this simplifies to:

$$|d\theta/dl| = \lim_{\Delta l \to 0} \left|\Delta\theta / \Delta l\right| = |\tan(\phi)/l|$$

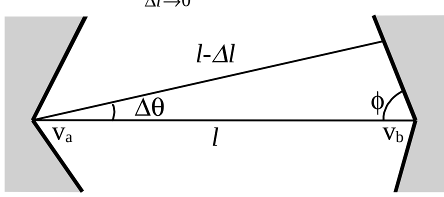


Figure 7: Deriving an expression for $|d\theta/dl|$.

For all 4 edges, we choose the one with ϕ closest to 90^o, which yields the maximum possible change in orientation. For this value of ϕ, the metric will be $|\tan(\phi)/l|$. We use this metric to rank v-grips.

7. Algorithm Implementation and Examples

Recall that the polygonal part is described by n vertices. For the polygonal part, we find $k \le n$ concave vertices flanked by straight edges in $O(n)$ time. We then consider each pair of concave vertices, checking the conditions in Theorems 1 and 2 in constant time. The result is a set of up to k^2 v-grips. Thus, all v-grips are found in $O(n + k^2)$ time. Computing the quality metric takes constant time for each v-grip and sorting requires $O(k^2 \log k)$ time as there are at most k^2 v-grips.

We implemented the algorithm in Visual BASIC. On a Pentium II 266 MHz PC running on Windows NT 4.0, the program execution time was under 0.02 seconds for a part with 30 vertices and 10 concave vertices, while interpreting. A Java implementation is available for online testing at http://alpha.ieor.berkeley.edu/vgrip.

Figure 8 shows a few pairs of vertices checked for being v-grips. Grips (a), (c) and (e) are v-grips, while (b) and (d) are not. Grip (d) is a case where the distance function is at a saddle point. Figure 9 shows a buckle and a glue-gun with two example grips each.

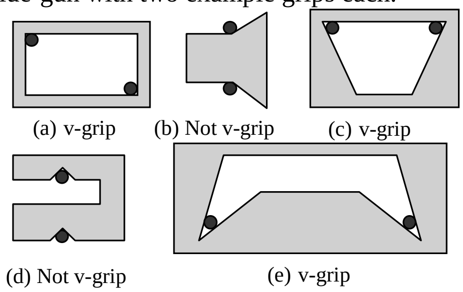


Figure 8: Examples of some pairs of vertices checked for being v-grips.

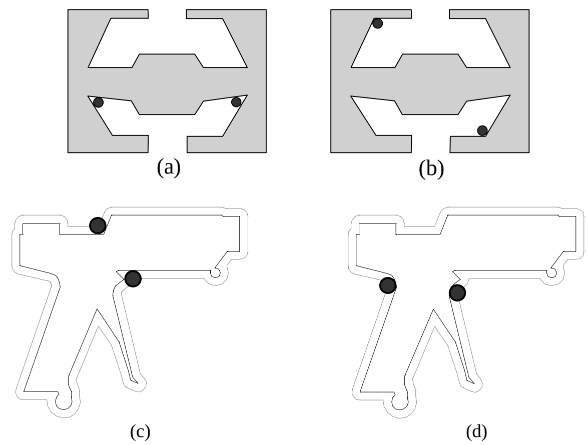


Figure 9: Example parts (buckle and glue gun) whose v-grips were found and ranked. The v-grip (a) is ranked better than (b) by the quality metric for the buckle. The v-grip (c) is ranked better than (d).

8. Jaws with Non-Zero Radii

If a jaw has a radius r, the part can be transformed by a Minkowsky addition, offsetting the polygons with a disk of radius r. As a result, points that were at a distance less than r from the original part lie on the interior of the transformed part. Thus, the transformed part's interior gives the positions of the jaw's center that result in a collision.

The transformed part with jaws of zero radii is an equivalent problem, as the lines of action do not change for any of the forces. (This transformation is also of interest in CAD/CAM for finding machine tool paths.) For a polygon (possibly with holes) described by n vertices, Held [8] gives a rigorous $O(n^2)$ algorithm that computes the Voronoi diagram and the resulting generalized polygons. Faster algorithms that run in $O(n \log n)$ exist in theory. From this output, we identify k accessible concave vertices on the original part. These vertices cannot be determined without the transforming the part, as the transformation changes part topology.

The transformed part is described by generalized polygons (edges can be either line segments or circular arcs), and can have a different topology from the original part. To avoid contact with convex vertices, we ignore the circular edges. An additional condition to check for v-grips with jaws of non-zero radii is that the jaws should not intersect.

9. V-grips for 3D Parts

In 3D, we assume the part sits on a planar work-surface under the influence of gravity. 3D v-grips are achieved with a pair of frictionless vertical cylinders closing monotonically and quasi-statically as shown in figures 10 and 1(c) and (d). In this section, we describe a numerical algorithm for computing all 3D v-grips for contracting v-grips of polyhedral parts where the jaws have zero radius and show how it can be applied to find critical shape parameters.

We are given a 3D polyhedral part model with the part's center of mass. The 3D algorithm repeatedly applies the 2D algorithm to projections of the 3D part. We define a *candidate 2D v-grip* as a 2D v-grip of the projection of the 3D part on the work-surface. Initially, when the part rests stably on the work-surface, the part's center of mass stays at a local minimum. Hence, the part does not rotate out of the plane until the distance function reaches a local minimum corresponding to a candidate 2D v-grip for that orientation.

We define a *candidate 3D v-grip* as a configuration of the part and the jaws such that the only feasible motion of the part is pure translation in a vertical direction along the jaws: gravity and the work-surface uniquely determine the part's configuration. We define a *3D v-grip* as a candidate 3D v-grip that results from a deterministic sequence of candidate 2D v-grips as the part is gripped.

First, we compute all stable orientations of the part on a flat horizontal work-surface by computing its convex hull and check all faces for stability based on the part's center of mass. For each stable orientation, we compute the planar projection of the part on the work-surface. For each projection, we compute all candidate 2D v-grips. For each of these, we compute the part's trajectory as we incrementally reduce the distance between the jaws. For each increment, we determine the local minimum of the center of mass' height near the previous configuration. In our algorithm, since the part rotates out of the plane only after a candidate 2D v-grip is achieved, the minimum height of the center of mass occurs only at candidate 2D v-grips. Hence, finding this minimum is reduced to a one-dimensional search over possible candidate 2D v-grips.

For any configuration in the part's trajectory, let $\mathbf{n_i}$ be wrenches caused by unit forces along the contact normals at each contact between the part and the jaw and let $\mathbf{N_j}$ be those at each contact between the part and the work-surface, all normals into the part. Let $\mathbf{W_h}$ be the subspace of the wrench space consisting of all wrenches caused by horizontal forces. Let $\mathbf{W_g}$ be the wrench caused by gravity. For each configuration in the trajectory, we check if one of the following conditions hold:

(1) A 3D v-grip is achieved if the origin of $\mathbf{W_h}$ is contained inside the convex hull of $\mathbf{n_i}$.

(2) A 3D equilibrium grip is achieved if the origin of $\mathbf{W_h}$ is on the boundary of the convex hull of $\mathbf{n_i}$. Note that this is not a 3D v-grip.

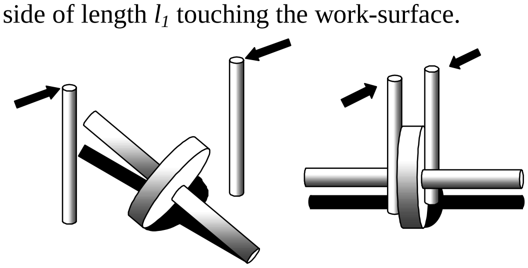
(3) The part falls away if $\mathbf{W_g}$ is not contained in the conical hull of $\mathbf{n_i}$ and $\mathbf{N_j}$.

If none of the three conditions are met, we find the next point in the part's trajectory by moving the jaws together incrementally and repeating the above.

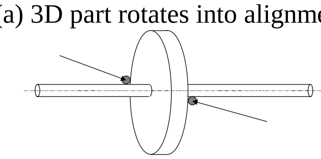
If the part has n vertices, and if k is the maximum number of concave vertices over all projections of the part on the work-surface, the number of candidate 2D v-grips is $O(nk^2)$. Each iteration in the algorithm requires $O(n^2)$ time. Thus, all 3D v-grips can be found in $O(n^3 k^2)$ time. The following examples illustrate the algorithm.

9.1 3D Example 1

Consider a contracting v-grip of the gear and shaft shown in figure 10. The cylindrical symmetry of the part reduces a degree of freedom making the results easier to visualize. The gear is a cylinder of radius R and thickness t. The shaft has a radius r, and has lengths l_1 and l_2 on either side of the gear. Initially, the part rests on the horizontal plane, as shown in figure 10(a) with the side of length l_1 touching the work-surface.



(a) 3D part rotates into alignment.



(b) Projection onto worksurface.

Figure 10: v-grip of a gear and shaft.

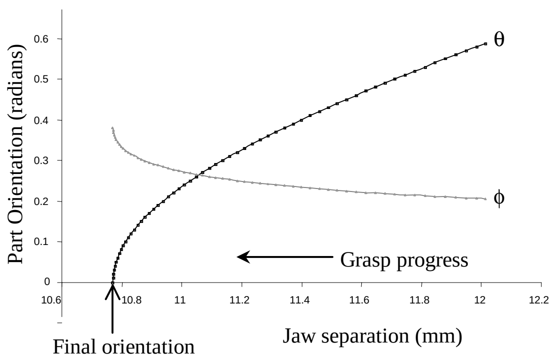
We describe the part's orientation by θ, the angle between the axis and the horizontal plane, and ϕ, the angle between the horizontal projection of the shaft and the line joining the jaws. Let w be the distance between the jaws. From the horizontal projection, it is clear that the only possible candidates for 2D v-grips are as shown in figure 10(b), and its mirror image.

Using theorem 2, we can compute the critical part dimensions for which there exists a 3D v-grip starting from this initial orientation as:

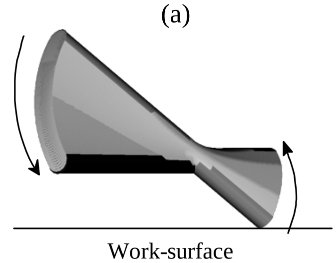
$$t < 2\sqrt{R^2 - r^2}\Big/\tan\theta$$

This is necessary and sufficient for first order immobility of the projection. As during gripping θ only decreases, it is enough to check this for the initial θ in the resting position.

Given θ, we can find w and ϕ from the horizontal projection. Hence, from w, we can uniquely determine θ and ϕ. Thus for a given w, the set of candidate orientations for points on the part's trajectory reduces to a singleton set making the minimization redundant. Termination occurs in the final orientation shown in figure 10(a) when a 3D v-grip occurs.



(a)



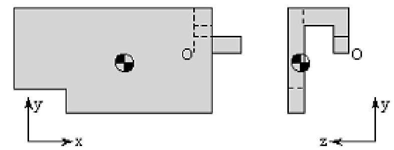
(b)

Figure 11: Orientation of the part as the jaws close. The parameters are: R=10, r=2, l_1=12, l_2=25, t=10. Jaw velocities are equal and opposite. In (b), the path of the shaft is traced as the grip aligns the part.

9.2 3D Example 2

As second example: consider the 3D part without axial symmetry shown in Figures 12 and 1(c) and (d). This part has a number of stable orientations when in contact with a planar work-surface under the influence of gravity. We study the gripping process starting with the stable part configuration shown in Figure 1(c) and show that the part will be aligned as the jaws are closed so that its largest planar surface is rotated to become parallel and aligned with the horizontal work-surface.

We analyze the part with a frame of reference whose origin is fixed at part vertex O. An orientation of the part is given by rotating the part from the orientation in figure 12(a) about the x-axis by θ, then about the y-axis by ϕ, and finally about the z-axis by φ. If we analyze the gripping in terms of the distance between the jaws and ignore the actual positions of the jaws, we can ignore the value of φ.



(a) Orthogonal top view (b) Orthogonal right view

Figure 12: Example 2: 3D Part without axial symmetry shown in figure 1(c) and 1(d).

Let the other two rotations be defined the transformation matrix $T(\theta, \phi)$. The conditions to prevent collision are: $[\{T(\theta, \phi)\,(v_{1c} - v_{2c}) \cdot \mathbf{e}_x\}^2 + \{T(\theta, \phi)\,(v_{1c} - v_{2c}) \cdot \mathbf{e}_y\}^2]^{0.5} = w$, where w is the distance between the jaws, and v_{1c} and v_{2c} are the 2D concave vertices where the jaws make contact. To model part motion, we must locally minimize the height of the center of mass given by max $[T(\theta, \phi)\,(g - v_i) \cdot \mathbf{e}_z]$, where g is the center of mass' position, over all vertices v_i of the polyhedron.

The result of the analysis of the v-grip is shown as a graph of $|\theta|$ and $|\phi|$ vs. w in Figure 13. Initially both θ and ϕ decrease until θ becomes zero. Then, θ stays constant while ϕ decreases to zero. Thus, the gripping process results in a 3D v-grip and aligns the part to the orientation where its largest planar surface becomes parallel and aligned with the horizontal work-surface.

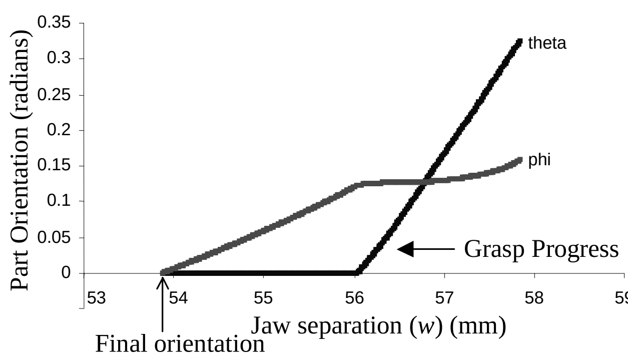


Figure 13: Orientation of part in figure 12 during the gripping process.

10. Discussion and Future Work

V-grips are an example of minimalist or RISC robotics as described in [4]. Parallel-jaw grippers are inexpensive, lightweight, and their small footprint facilitates access for industrial applications. Algorithms that automatically compute grip configurations can be particularly useful during the design stage, when small changes in part geometry can greatly facilitate part handling during assembly and manufacture.

In this paper we define v-grips and a new quality metric for ranking them based on sensitivity to infinitesimal disturbances. This metric can be computed quickly and is consistent with intuition. It can be of particular use in the design stage to ensure that high accuracy is easily attained during manufacturing. For a polygonal part with polygonal holes, we give a fast algorithm for computing and ranking all v-grips, characterize its complexity, and report on an implemented version. We extend the definition to jaws with non-zero radii. We define 3D v-grips with a pair of frictionless vertical cylinders, give a numerical algorithm for computing all 3D v-grips, and illustrate with examples.

In future work, we will explore alternatives to the quality metric in Section 6, which takes into consideration only the local shape around the jaws. An alternative metric may be based on a measure of the "capture region": the volume of C-space that is guaranteed to converge to the desired grip. We will also extend the definition of v-grips to curved parts and more general jaw shapes. We are also developing a model of part deformation resulting from v-grips.

11. Acknowledgements

We thank Brian Carlisle and John Craig for ongoing collaboration in grip analysis for industrial applications, Yong Liu for early contributions and Gary Bone, Martin Held, Tao Zhang, Mark Moll, Ron Alterovitz and K. Subramanian for valuable feedback.

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